Q1.

(a) $2\csc 2t = \sec t \csc t$.

$$2\csc 2t = \frac{2}{\sin(2t)}$$

$$\sin(2t) = 2\sin t \cos t.$$

$$= \frac{2}{2\sin t \cos t} = \frac{1}{\sin t \cos t}.$$

But $\dfrac{1}{\sin t \cos t} = \sec t \csc t$

True

(b) $\cot^2\theta + \sec^2\theta = \tan^2\theta + \csc^2\theta$

$\cot^2\theta + \sec^2\theta$

$\sec^2\theta = 1 + \tan^2\theta$

$= 1 + \cot^2\theta + \tan^2\theta.$

$= 1 + \cot^2\theta = \csc^2\theta.$

$= \csc^2\theta + \tan^2\theta$

True

(c) $\sin x \sin 2x + \cos x \cos 2x = \cos x$

$\sin x \sin 2x + \cos x \cos 2x = \cos x.$

Consid $\cos x = \cos(2x - x).$

$\cos(\theta \pm \phi) = \cos\theta \cos\phi \mp \sin\theta \sin\phi$

$\cos(2x - x) = \cos x \cos 2x + \sin x \sin 2x$

True

Scanned with CamScanner

Question 2

(a) $f(x) = \dfrac{1 - e^{x^2}}{1 - e^{1-x^2}}$

For $f(x)$ to be defined, the expression

$$\frac{1-e^{x^2}}{1-e^{1-x^2}} \geq 0$$

$$1 - e^{x^2} \geq 0 \quad (1 - e^{1-x^2}) \geq 0.$$

$$1 - e^{x^2} \geq 0$$

$$e^{x^2} \geq 1$$

$$\log e^{x^2} \geq \log 1.$$

$$\log(e)\, x^2 \geq 0.$$

$$x^2 \geq \frac{0}{\log e}$$

$$x^2 \geq 0$$

$$x \geq 0$$

Domain of $f(x)$ is $(0, \infty)$.

b) $f(x) = \dfrac{1+x}{e^{\cos x}}.$

$$\frac{1+x}{e^{\cos x}} \geq 0$$

$$1 + x \geq 0$$

$$x \geq -1.$$

Domain is $(-1, \infty)$.

(f) $g(t) = \sqrt{10^t - 100}$

$10^t - 100 \geq 0.$

$10^t \geq 100$

$\log 10^t \geq \log 100$

$t \log 10 \geq \log 100$

~~td~~ $t \geq \frac{\log 100}{\log 10}$

$t \geq 2$

Domain is $(2, \infty)$.

Q3

(a) $\lim_{x \to -\infty} (x^2 + 2x^7).$

$\lim_{x \to -\infty} (x^2 + 2x^7) = \lim_{x \to -\infty} [x^2 [1 + 2x^5$

(b) $\lim\limits_{x \to \infty} \left[\ln(2+x) - \ln(1+x) \right].$

$$\lim_{x \to \infty} \left[\ln(2+x) - \ln(1+x) \right] = \lim_{x \to \infty} \left[\ln\left(\frac{2+x}{1+x} \right) \right)$$

$$= \ln \left[\frac{2+\infty}{1+\infty} \right] = \infty$$

$$= \underline{\underline{\infty}}.$$

(c) $\lim\limits_{x \to \infty} \left[\ln(1+x^2) - \ln(1+x) \right].$

$$\lim_{x \to \infty} \left(\ln \left[\frac{1+x^2}{1+x} \right] \right)$$

$$= \ln \frac{1+\infty^2}{1+\infty} = \infty$$

$$= \infty.$$

(d) $\lim\limits_{x \to 0^+} \tan^{-1}(\ln x).$

If $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = L$ thin $\lim_{x \to a} f(x) = L.$

$$\lim \tan^{-1}(\ln x) = -\frac{\pi}{2}$$

<u>Q4</u>

(a) $y^2(6-x) = x^3, (2, \sqrt{2})$.

Implicitly differentiating both sides with respect to x taking 'y' as a function of x.

$$6y^2 - xy^2 = x^3$$

$$\cancel{6y^2 - xy^2 - x^3 = 0}.$$

$$\frac{\partial}{\partial x}(2x^2 - xy^2) = \frac{\partial}{\partial x}(x^3).$$

$$4x - \left(x\frac{\partial y}{\partial x} + \frac{\partial x}{\partial x}y^2\right) = 3x^2.$$

$$4x - x\frac{\partial y}{\partial x} + y^2 = 3x^2.$$

$$\cancel{-x\frac{\partial y}{\partial x}}\ 4x - 3x^2 + y^2 = x\frac{\partial y}{\partial x}.$$

$$\frac{\partial y}{\partial x} = \frac{4x - 3x^2 + y^2}{x}.$$

$$\left.\frac{\partial y}{\partial x}\right|_{(2,\sqrt{2})} = \frac{4(2) - 3(2)^2 + (\sqrt{2})^2}{2} = \frac{8 - 12 + 2}{2} = -1$$

Slope $= -1$

Equation of tangent is

$$y - y_1 = m(x - x_1)$$

$$y - \sqrt{2} = -1(x - 2)$$

$$y - \sqrt{2} = -x + 2.$$

$$y = -x + 2 + \sqrt{2}$$

(b) $x^2 + 2xy + 4y^2 = 12, \ (2,1).$

$$\frac{\partial}{\partial x}(x^2 + 2xy + 4y^2) = \frac{\partial}{\partial x}(12).$$

$$2x + 2\left(\frac{\partial y}{\partial x}x + \frac{\partial x}{\partial x}y\right) + 4\frac{\partial y^2}{\partial y} \times \frac{\partial y}{\partial x} = 0.$$

$$2x + 2\left(y + x\frac{\partial}{\partial x}(y)\right) + 8y\frac{\partial}{\partial x}(y) = 0.$$

$$2x + 2y + 2x\frac{\partial}{\partial x}(y) + 8y\frac{\partial}{\partial x}(y) = 0.$$

$$\cancel{\frac{\partial y}{}}\ \frac{\partial}{\partial x}(y)\left[2x + 8y\right] = 2x + 2y.$$

$$\frac{\partial y}{\partial x} = \frac{2x+2y}{2x+8y} = \frac{2(x+y)}{2(x+4y)} = \frac{x+y}{x+4y}$$

$$\left.\frac{\partial y}{\partial x}\right|_{(2,1)} = \frac{2+1}{2+4(1)} = \frac{3}{6} = \frac{1}{2}. = \text{Slope}$$

~~Tangent of th~~ Equation of tangent

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{1}{2}(x - 2)$$

$$y - 1 = \frac{1}{2}x - 1$$

$$y = \frac{1}{2}x - 1 + 1 = \frac{1}{2}x.$$

$$y = \frac{1}{2}x.$$

Scanned with CamScanner

Q5

(a) $f(s) = \ln \ln s$

$$\frac{\partial}{\partial x} \ln \ln s$$

chain rule. $\frac{\partial y}{\partial x} = \frac{\partial y}{\partial u} \cdot \frac{\partial u}{\partial x}$.

$$\frac{\partial}{\partial x} \ln \ln s = \frac{1}{\ln(s)} \frac{\partial}{\partial x} \ln(s).$$

$$\frac{\partial}{\partial s} \ln(s) = \frac{1}{s}.$$

$$= \frac{1}{\ln s} \times \frac{1}{s} = \frac{1}{s \ln s}.$$

$$= \frac{1}{s \ln s}.$$

b) $f(\theta) = \cos(\theta^2)$.

$$= \frac{\partial}{\partial \theta} \cos(\theta^2) \; . = \frac{\partial}{\partial \theta} \cos \theta^2 \frac{\partial}{\partial \theta} \theta^2 , \; \frac{\partial}{\partial \theta} \cos\theta = -\sin \theta$$

Apply chain rule.

$$= -\sin(\theta^2) \frac{\partial}{\partial x} (\theta^2).$$

$$\frac{\partial}{\partial x}(x^2) = 2\theta.$$

$$= -\sin(\theta^2) \cdot 2\theta$$

$$= -2\theta \sin \theta^2$$

Scanned with CamScanner

(c) $f(t) = \frac{1}{(2t+1)^2} = (2t+1)^{-2}$.

Let $2t+1 = u$.

$= \frac{\partial}{\partial u} u^{-2}$. Apply chain rule $\frac{\partial y}{\partial x} = \frac{\partial y}{\partial u} \cdot \frac{\partial y}{\partial x}$.

$= -2u^{-3} \cdot \frac{\partial}{\partial t}(u)$, $\frac{\partial}{\partial t} u = \frac{\partial}{\partial t}(2t+1) = 2$.

$= -2(2t+1)^{-3} \cdot 2 = \frac{-4}{(2t+1)^3}$

$= \frac{-4}{(2t+1)^3}$

(d) $f(t) = 2t^3$

$= \frac{\partial}{\partial t}(2t^3) = 2\frac{\partial}{\partial t} t^3$.

$= 2 \cdot (3t^{3-1}) = 2(3t^2) = 6t^2$

$= 6t^2$.